

# The universal quantum driving force to speed up a quantum computation — The unitary quantum dynamics

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## Abstract

On the basis of the polynomial-time unstructured quantum search process it is shown that the unitary quantum dynamics in quantum mechanics is the universal quantum driving force to speed up a quantum computation. This assertion supports strongly in theory that the unitary quantum dynamics is the fundamental and universal principle in nature. On the other hand, the symmetric structure of Hilbert space of a composite quantum system is the quantum-computing resource that is not owned by classical computation. A new quantum-computing speedup theory is set up on the basis of the unitary quantum dynamics in a universal quantum computational model. Both the unitary quantum dynamics and the symmetric structure and property of the Hilbert space are mainly responsible for an exponential quantum-computing speedup for a general efficient quantum algorithm. The inherent importance for the unitary quantum dynamics to speed up a quantum computation lies in the unique ability of the unitary quantum dynamics to build the effective interaction between the symmetric structure of the Hilbert space (or the fundamental quantum-mechanical principles) and the mathematical symmetric structure (or the mathematical logic principles) of a problem to be solved on the quantum system. This unique ability is not owned by the reversible classical mechanics and the reversible equilibrium-state thermodynamics. It may result in an essential difference of computational power between quantum computation and classical computation by combining the symmetric structure and property of the Hilbert space. In theory and experiment this effective interaction may be built with the help of the unitary manipulation on the mathematical-logic functional operations of the problem. The quantum-computing speedup theory also provides reasonable mechanisms for exponential quantum-computing speedup for the existing efficient quantum algorithms that are constructed in the frame of the quantum parallel principle. These existing quantum algorithms including the hidden-subgroup-problem quantum algorithms and conventional quantum search algorithms have the common character that the symmetric structure and property of the Hilbert space does not have any effective effect on these quantum algorithms. This could be the main reason why these quantum algorithms including the efficient ones are quite special and considered to be semiclassical.

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## 1. Introduction

Quantum computation is based on both quantum mechanical principles and mathematical logic principles. It employs quantum mechanical principles to study computational science. Its theoretical basis is quantum physics. Therefore, quantum computation is essentially different from classical computation based on classical physics. At present its influence may reach far beyond the quantum computational science. It has already a great influence on the physical science. It could revolutionize the future computational science and technology. It will have an extensive influence on many other science disciplines in future. Quantum computation is mainly referred to mathematical-sense universal quantum computation. It also may contain quantum simulation which is usually purely quantum mechanical. Historically the emergence of quantum computation is related to classical computation and especially reversible classical computation. This may be seen in the historical evolution from reversible classical computation to universal quantum computation. A classical computation obeys the classical physical laws. It also is required to obey the mathematical-logic principles. These are the theoretical basis for all digital computers today. It is well known that a classical computation is irreversible in mathematical logic, but time evolution process of the classical physical system to execute a computation is also irreversible. Due to both the irreversibilities an irreversible classical computation can generate a large amount of heat in a high-speed computer today [1]. This heating is a severe limitation for a classical computer to operate in a very high speed. In order to resolve the heating problem the reversible classical computational models had been proposed in the middle of the last century [2]. The basic idea for the reversible computational models is that every computational step is made mathematical-logically reversible. That the mathematical-logic operations are made reversible is not the final scheme to resolve thoroughly the heating problem for a high-speed classical computer, because the time evolution process of a classical physical system to execute a computation still could be irreversible. For example, the SWAP operation is reversible in mathematical logic, but it can be realized directly on an irreversible classical computer. One might employ an ideal reversible process of macroscopic thermodynamics [2a] or classical mechanics [2c] to realize a reversible computation, so that both the mathematical-logic reversibility and the physical-process reversibility could be achieved in computation at the same time. But such an ideal physical process is generally hard to realize in practice. On the other hand, it is well known that according to quantum mechanics a microscopic physical system obeys naturally the unitary quantum dynamics, i.e., the Schrödinger equation. When a computation is carried out in a microscopic quantum physical system, the computational process obeys naturally the unitary quantum dynamics and hence it is naturally reversible. Thus, the heating problem could be resolved thoroughly by employing the unitary operators of a microscopic physical system to realize the reversible mathematical-logic operations in a computational process. Both the quantum Turing machine model [3] and the universal quantum Turing machine model [4] use the unitary operators to realize a computational process.

Therefore, there is not any heating problem in both the computational models.

Now every irreversible mathematical logic operation of a classical computation is made reversible according to the reversible classical Turing machine model [2a, 2b]. Then the reversible mathematical logic operation is replaced with the discrete unitary operation according to the unitary quantum Turing machine model [3] so that the physical process of the computation is made unitary, while at the same time keeping every mathematical logic operation reversible. This shows that both the reversible classical computational models [2] and the quantum Turing machine models [3] together may ensure that a computational process obeys the discrete unitary quantum dynamics (in physical process) and is also compatible with the mathematical logic principles in a reversible or unitary form (in computation). However, both the reversible classical computational models [2] and the quantum Turing machine models [3] are not thought of as universal quantum computational models.

Since the early 1980s the main research on the quantum computational models and especially the universal quantum computational models which include the universal quantum Turing machine [4] and the universal quantum circuit model [5] has focused on investigation of the computational performance and especially the computational power. Though the heating problem could be resolved in the reversible classical computational models [2] and the quantum Turing machine models [3], it could also be resolved by using other schemes. For example, by improving cooling techniques or lowering the energy consuming in computation the heating problem could be resolved as well. Therefore, the computational performance becomes a key problem to a universal quantum computer. Whether or not someday a quantum computer can replace a classical computer is greatly dependent upon its computational performance. It has been believed extensively [4] that every finitely realizable physical system can be simulated efficiently by a universal quantum Turing machine (the quantum Church-Turing thesis). Both a universal quantum Turing machine model and a universal quantum circuit model are equivalent to each other in quantum computation [6]. These mean that in order to investigate the computational performance of a quantum computer one needs only to investigate the computational performance of a universal quantum computational model [4, 5]. It also has been believed extensively [8] that it is essentially difficult to simulate efficiently a quantum physical system on a classical computer. This implies that a quantum computational model which is based on the quantum mechanical principles could not be simulated efficiently by a classical computational model. Both the reversible computational models [2] and the quantum Turing machine models [3] can be simulated efficiently by a classical Turing machine. Therefore, while the reversible computational models are clearly classical, the quantum Turing machine models still could be classical computationally in the sense that they can be efficiently simulated by a classical Turing machine, and they are not a universal quantum computational model. A universal quantum computational model [4, 5, 6, 7] is based on quantum mechanics. According to quantum mechanics [9] a closed quantum system is described completely by the quantum states of the Hilbert state space of the quantum system and

its time evolution process obeys the unitary quantum dynamics. Because a universal quantum computational model is based on quantum mechanics, the time evolution process of the quantum system to execute a quantum computation obeys the unitary quantum dynamics, while the quantum system itself is described completely by its quantum states. Therefore, a universal quantum computational model still obeys the unitary quantum dynamics and may make use of a variety of fundamental properties of quantum states of the quantum system in a computation. These fundamental quantum-state properties include the quantum superposition principle, the quantum coherence interference effects, and also include the properties of the special quantum states such as the quantum entanglement states, the multiple-quantum coherence mixed states, and so on. It seems that there is not any difference between a universal quantum computational model and quantum mechanics. In fact, there must be a constraint on a universal quantum computational model so that it can be thought of as a computational model instead of quantum mechanics. This constraint is that a universal quantum computational model must be compatible with mathematical-logic principles, as emphasized explicitly in Refs. [21a, 29]. This leads to that the interaction becomes inevitable between the mathematical logic principles that a mathematical or computational problem obeys and the fundamental quantum-mechanical principles that a universal quantum computer obeys. Mathematical-logic principles that a problem obeys are naturally carried out by the fundamental mathematical-logic operational rules in a computation [2, 3, 4, 5, 6, 7, 31]. They neither belong to the classical physics nor the quantum physics, that is, they are independent of both the classical and quantum physics. Therefore, a universal quantum computational model must obey the three fundamental principles (or properties): (i) the physical process of a quantum computation obeys the unitary quantum dynamics, (ii) a quantum computation is compatible with the mathematical logic principles in a reversible or unitary form, meaning that the quantum computation obeys the mathematical logic principles in a reversible or unitary form, (iii) the quantum system to execute a quantum computation is described completely by its quantum states and the quantum computation allows to make use of a variety of fundamental quantum-state properties of the quantum system. Some of these fundamental principles (or properties), that is, the unitary quantum dynamics, the mathematical-logic principles, and the fundamental quantum-state properties are believed to be mainly responsible for the computational power of a universal quantum computer that can outperform essentially a classical computer. Since the mathematical-logic principles (including the mathematical symmetric structures, etc., of course) are independent of a detailed physical computational model, they are considered not to have an essential effect on the computational power of a quantum computer. A similar viewpoint [4, 5] that pure mathematics does not affect essentially computational performance of a computer also was pointed out in the past. Therefore, only the fundamental principles (or properties) in quantum mechanics may have a possibility to make an essential influence on the computational power of a universal quantum computer and especially to be responsible for an exponential quantum-computing speedup. According to

quantum mechanics there may be two essentially different quantum-computing speedup theories. One is based on the fundamental quantum-state properties. This is the earliest quantum-computing speedup theory — the quantum parallel principle [4] which is based on the quantum-state superposition principle in quantum mechanics. Another is based on the unitary quantum dynamics [21, 32]. Though mathematical-logic principles are not essentially responsible for a quantum-computing speedup, they play a key role in judging which one of the two quantum-computing speedup theories to be responsible essentially for a quantum-computing speedup. The unitary quantum dynamics has been overlooked in studying mechanisms of quantum-computing speedup in the past decades. Its inherent importance to speed up a quantum computation was not revealed until these extraordinarily fast quantum algorithms [21, 32] are discovered by the present author in the early 2000s. It is this inherent importance (See the comment [33]) that ultimately leads the present author to put the unitary quantum dynamics at the center of the universal quantum computational models and suggest it as the fundamental and universal quantum-mechanical principle in nature.

In the past decades the computational performance of a quantum computer has been extensively investigated. It was shown [10, 3] that the reversible classical computational models and the quantum Turing machine models can be as powerful as the irreversible classical computational model. Feynman [8] had paid attention to the known fact that simulating a quantum system is essentially difficult on a classical computer, and more importantly he suggested that this difficulty could be avoided with the help of the principles of quantum mechanics. Feynman's idea implies that a quantum computer might be more powerful than a classical computer. An important and essential advance came soon after the universal quantum Turing machine model [4] was set up. Deutsch proposed for the first time a quantum algorithm according to the quantum parallel principle [4] and then Deutsch and Jozsa [11] generalized the quantum algorithm to show that a quantum computer indeed can solve a problem in polynomial time, while a deterministic classical computer can not solve the same problem in polynomial time. This result also was further confirmed and developed [12]. This is the first remarkable example that the quantum parallel principle based on the superposition principle of quantum mechanics plays an important role for a quantum algorithm outperforming essentially its classical counterpart. Soon after the remarkable work [11] several important quantum algorithms [7, 13, 14] were discovered also based on the quantum parallel principle. They further show that a quantum computer even can outperform essentially a probabilistic classical computer. Since then, the quantum parallel principle is considered as a fundamental rule to guide the design of a quantum algorithm. Many quantum algorithms [11, 12, 13, 14, 15, 16] have been discovered that can outperform essentially their classical counterparts under the guidance of the quantum parallel principle in the past two decades. Among these quantum algorithms the polynomial-time quantum algorithms for the prime factorization and discrete logarithms [14] are one of the important applications of the quantum parallel principle. These quantum algorithms show that the quantum parallel principle

indeed makes a great contribution to development of the quantum computational science in the past two decades. However, at present it is believed that the quantum parallel principle is not enough powerful to solve many important problems such as the graph isomorphism problem and most non-Abelian hidden subgroup problems [16]. It is believed extensively that these non-Abelian hidden subgroup problems are not harder than the NP-complete problems [17] in computational complexity.

Since the oracle-based or black-box-based quantum algorithms [11, 12, 7, 13] were first formulated, a large number of oracle-based quantum algorithms have been discovered to solve the specific problems on a quantum computer. An unstructured search problem could be the most important one of these problems. An unstructured quantum search algorithm has an extensive application in practice. In particular, it may be used to solve an NP-complete problem. It is well known that most important problems in computational science are either polynomial-time or NP-complete. The oracle-based quantum search algorithms have been investigated extensively in the past decades. The conventional (unstructured) quantum search algorithm (i.e., the Grover's quantum search algorithm [23]) is the first oracle-based quantum search algorithm with a square speedup. It is based on the amplitude-amplification mechanism [23, 24]. Another important quantum search algorithm is the adiabatic quantum search algorithm [25]. It is based on the continuous-time adiabatic state-transfer mechanism in a space-independent quantum system [34]. Though these quantum search algorithms are discovered also under the guidance of the quantum parallel principle, their computational power is far more weak than an exponential speedup in worst case. Actually, it has been shown that the conventional quantum search algorithm [23, 24] is square speedup and moreover this square speedup is optimal [26], and the adiabatic quantum search algorithm is also square speedup in worst case [27]. More generally, it has been shown that any oracle-based quantum algorithm to compute a total Boolean function can achieve only a polynomial speedup over its classical counterpart [28]. Therefore, all these oracle-based quantum search algorithms are not enough powerful to solve efficiently the NP-complete problems.

The quantum simulation was initiated by the Feynman's work [8]. When the mathematical-logic principles are not considered explicitly, the universal quantum circuit model [5] is reduced to the quantum simulator. A usual quantum simulation usually does not consider explicitly the mathematical-logic principles. Thus, it is usually thought of as a purely quantum-mechanical process instead of a mathematical-sense quantum computation. This results in that a number of important problems in quantum computation can not be owned by a conventional quantum simulation. Many fundamental and essential problems of quantum computation can not be solved by studying alone a conventional quantum simulation. Based on the Feynman's ideas several important works [18] on the quantum simulation have been developed in detail. These works show that many quantum systems may be simulated efficiently on a quantum simulator, while they could not be simulated efficiently on a classical computer or it has not yet been found that there are efficient algorithms to simulate these

quantum systems on a classical computer. It has been shown [18d] that the quantum parallel principle still plays an important role for the quantum simulation outperforming the classical simulation. The Trotter-Suzuki formalism [19, 20] could provide a unified frame to describe both a conventional quantum simulation and a classical simulation. It has been shown that it is hard to solve an unstructured search problem by the quantum simulation [55]. Though the special unitary quantum simulation may be used to solve an unstructured search problem in a quantum spin system (or ensemble) [21], it has not yet been proven that it is both efficient and scale. It also has been shown [22] that most unitary transformations can not be simulated efficiently on a quantum computer. Hence the quantum simulation could not be enough powerful to solve efficiently an unstructured search problem.

It has been believed extensively that an NP-complete problem can not be solved efficiently on a classical computer [17]. It has turned out that a quantum computer is more powerful than a classical computer. Then one of the most important problems in the quantum computational science is to answer the question: whether or not a universal quantum computer is capable of solving an NP-complete problem in polynomial time. As described in the previous paragraphs, a number of efficient quantum algorithms have been discovered in the past two decades and most of them are discovered in the frame of the quantum parallel principle. A detailed review for a variety of quantum algorithms including those mentioned above, the topological quantum algorithms, the random-walk-based quantum algorithms, etc., may be seen in recent review papers [15d, 15e]. However, no one of these quantum algorithms can show that a universal quantum computer is able to solve an NP-complete problem in polynomial time.

Indeed, in the past decades the quantum parallel principle achieves a great success in guiding construction of the efficient quantum algorithms, but after examining these existing quantum algorithms, as described above, one may find that only some special quantum algorithms can achieve an exponential quantum-computing speedup, which include mainly the Abelian hidden-subgroup-problem (HSP) quantum algorithms [11 – 15] and some non-Abelian HSP quantum algorithms [16, 15d, 15e]. This shows that the quantum parallel principle is greatly limited. A question therefore arises whether or not the quantum parallel principle is really responsible for an essential quantum-computing speedup in quantum computation. This question will be answered in the paper. This paper is mainly devoted to studying mechanisms of quantum-computing speedup and especially exponential quantum-computing speedup in quantum computation. It investigates how the fundamental quantum-mechanical principles and properties affect computational power of a universal quantum computer. The research in the paper is based on the present author's works in the past decade and particularly the polynomial-time unstructured quantum search processes [56].

## 2. The Symmetric structure and property of direct-product Hilbert space and the polynomial-time unstruc-

## tured quantum search processes

A quantum algorithm may be constructed generally in the frame of a universal quantum computational model, but it may be realized in various quantum systems. It is also possible that a quantum system could be better to realize a quantum computation than other quantum systems. It seems that difference in computational power for these quantum systems could not be essential according to the quantum Church-Turing thesis [4]. However, a universal quantum computational model [4, 5, 6, 7] generally does not consider explicitly whether or not the symmetric structure and property of the Hilbert space of a quantum system to execute a quantum computation may affect essentially quantum computational performance. Here the Hilbert space may be referred to the Hilbert state space or its corresponding operator space of a quantum system (the operator space also is often called the Liouville operator space in a quantum ensemble). Obviously, a universal quantum computational model allows ones to make use of the symmetric structure and property of the Hilbert space to improve the quantum computational performance [21, 29]. For example, the symmetrical structure and property of the multiple-quantum operator algebra space [35] has been used to simplify the unitary quantum simulations in quantum spin systems or ensembles (See also the present author's early works [36, 37]). The symmetric structure of the Hilbert space of a quantum system is another fundamental attribute of quantum mechanics which is different from the unitary quantum dynamics and the fundamental quantum-state properties. It is the quantum-computing resource that is not owned by classical computation. It could affect essentially quantum computational performance. In quantum mechanics [9] this fundamental attribute is closely related to the basic postulate that the Hilbert space of a composite quantum system is a direct product of the Hilbert spaces of the component systems of the composite system. Here it must be emphasized that symmetric structure of the Hilbert space of a quantum system is different in concept from pure mathematical symmetric structure of a problem to be solved. The former belongs to quantum physics, while the latter is of pure mathematics and does not have an essential effect on quantum computational performance. Then in quantum computation there are the two essentially different symmetric structures. One of which is of quantum system (or quantum computer) and another of a problem to be solved [21a]. In quantum computation one must explicitly distinguish the two essentially different symmetric structures from each other. As a typical instance, in quantum computation a hidden subgroup (HS) problem [13, 14, 15, 16] may be defined according to the symmetric structure of a specific group, e.g., an Abelian group or a non-abelian group, but it is still a pure mathematical problem. Its group symmetric structure is independent of any detailed physical computational model. Hence it is different from any symmetric structure of the Hilbert space of the quantum system that is used as a quantum computer to solve the HS problem. It does not affect essentially quantum computational performance of the quantum computer. Another typical example is a structured search problem in quantum computation. The mathematical symmetric structure of a structured search problem is different from any symmetric structure of the Hilbert space of the quantum system



used to solve the search problem. It does not affect essentially the quantum-searching speedup. Thus, just like an unstructured quantum search algorithm, a structured quantum search algorithm only can achieve a square speedup with respect to its classical counterpart [38]. Researchers could confuse the mathematical symmetric structure of a mathematical or computational problem with that one of the Hilbert space of the quantum system used to solve the problem. Sometimes this is because the two symmetric structures are in the same Hilbert space in quantum computation. A typical example may be seen in the conventional unstructured quantum search algorithm [23, 24], where the search space of the unstructured search problem is directly taken as the Hilbert space of the  $n$ -qubit quantum system used to solve the search problem. It has been extensively investigated [13, 14, 15, 16] how a quantum computer could solve efficiently the HS problems that have the specific group symmetric structures. However, here it is investigated how the symmetric structure and property of the Hilbert space of a quantum system may affect or even improve essentially computational performance of any quantum computational process running in the quantum system to solve a problem that could have some purely mathematical symmetric structure. It is clear that the two investigations are two completely different things.

A conventional quantum search algorithm usually works in an  $n$ -qubit spin system. Such a spin system has a typical tensor product (or direct product) symmetric structure in its Hilbert space. It has been shown that such a tensor-product symmetric structure is not necessary to achieve a square speedup for a conventional quantum search algorithm [39]. That is, the square speedup has nothing to do with the symmetric structure of the tensor-product Hilbert space of the  $n$ -qubit spin system. However, it is shown below that the symmetric structure of the tensor-product Hilbert space is necessary for a polynomial-time unstructured quantum search process. In a usual quantum search algorithm the search space for an unstructured search problem is usually taken as the Hilbert space of a quantum system. From the point of view of pure mathematics the search space is unstructured, while from the point of view of quantum mechanics the Hilbert space that serves as the search space may be structured or unstructured, which is dependent on the quantum system. Here researchers should pay attention to the fact that the symmetric structure of the search space may be different from that one of the Hilbert space. The former originates from the search problem or the search algorithm, while the latter is of the quantum system (or the quantum computer). However, if one ignores any possible symmetric structure of the Hilbert space, then the Hilbert space may be treated as an unstructured space even if it is a tensor-product Hilbert space of an  $n$ -qubit spin system. This means that in quantum computation there is no way to reduce the exponentially large search space [54] to a polynomially small search space [54] for an unstructured search problem and there is no way to reduce the unstructured quantum search process to a structured one unless the symmetric structure and property of the Hilbert space is considered explicitly. The conventional quantum search algorithms [23, 24] and the adiabatic quantum search algorithms [25] do not consider explicitly the symmetric struc-

ture and property of the Hilbert space. They have an exponentially large search space, resulting in that they cannot achieve an exponential quantum-searching speedup. In fact, a conventional quantum search algorithm [23, 24] can achieve only a square speedup [26] and the adiabatic quantum search algorithm [25] also achieves only a square speedup in the worst case [27]. Therefore, there is no way to improve essentially these quantum search algorithms (See, Refs. [26, 27]) unless the symmetric structure and property of the Hilbert space is considered explicitly. Furthermore, it will be seen later that even if the symmetric structure and property of the Hilbert space of an  $n$ -qubit spin system is considered explicitly, these usual quantum search algorithms [23, 24, 25] that work in the spin system could not be improved essentially.

If there exists the proper symmetric structure and property in the Hilbert space, then there exists a possibility that the symmetric structure and property of the Hilbert space could affect essentially an unstructured quantum search process and could be exploited to speed up the quantum search process [21, 29]. One might image intuitively that the unstructured search process could be reduced to some structured quantum search process due to the effect of the symmetric structure and property of the Hilbert space on the unstructured search process, resulting in that the unstructured search process is sped up. The problem is that the symmetric structure and property of the Hilbert space is independent of any mathematical logic principles including the mathematical symmetric structure of the unstructured search problem. How can the symmetric structure and property of the Hilbert space affect the unstructured search process? There must exist the quantum-mechanical principle to bring together both the symmetric structure and property of the Hilbert space and the mathematical logic principles of the search problem. Otherwise the symmetric structure and property of the Hilbert space could not affect essentially the quantum search process. This fundamental quantum-mechanical principle is the unitary quantum dynamics. The importance inherent in the quantum-computing speedup for the unitary quantum dynamics is that the unitary quantum dynamics is able to build the effective interaction between the symmetric structure and property of the Hilbert space of a quantum system and the mathematical logic principles of a problem to be solved on the quantum system. Whether or not this possibility that the symmetric structure and property of the Hilbert space could affect essentially an unstructured search process can become real is completely dependent on the unitary quantum dynamics. Therefore, there are the two fundamental quantum-mechanical principles to help an unstructured quantum search process to bypass the square speedup limitation. One of which is the unitary quantum dynamics in time and space. Another is the symmetric structure and property of the Hilbert space of a quantum system. In the past decade around the present author has investigated extensively how the two fundamental quantum-mechanical principles can improve essentially the computational performance of a quantum search process. These two fundamental quantum-mechanical principles lead to that there are polynomial-time unstructured quantum search processes in quantum computation [56]. These quantum search processes may be used to solve the  $NP$ -complete problems in polynomial time, indicating that in computational

complexity there is the relation  $\mathbf{NP} = \mathbf{P}$  on a universal quantum computer. A polynomial-time quantum search process [56] consists of the two different parts in structure. The first part [21, 29, 37, 35] is to realize dynamically an efficient reduction from the exponentially large Hilbert space that serves as the unstructured search space to the polynomially small state subspace (or state subset) of the Hilbert space in which the quantum states carry the information of the component states of the marked state of the unstructured search problem. This part is realized on the basis of both the unitary quantum dynamics and the symmetric structure and property of the tensor-product Hilbert space. The second part is to realize an exponential quantum-state-difference amplification. It is really an inverse process of the unitary dynamical state-locking process [30]. An exponential quantum-state-difference amplification usually could be realized in the time- and space-dependent quantum system of a single atom motioning in time and space. Therefore, the second part is realized on the basis of the unitary quantum dynamics in time and space.

In a polynomial-time quantum search process the symmetric structure and property of the tensor-product Hilbert space of an  $n$ -qubit (or more generally  $n$ -partite) composite quantum system has to be considered explicitly, while this consideration is not necessary in a conventional quantum search algorithm. This leads to that there is an essential difference in theory between a polynomial-time quantum search process and both the conventional quantum search algorithms [23, 24] and the adiabatic quantum search algorithm [25]. A usual quantum search algorithm [23, 24] may be generally written as

$$|\Psi_f^S\rangle = U_k C_S(\theta_k) U_{k-1} C_S(\theta_{k-1}) \dots U_1 C_S(\theta_1) |\Psi_0\rangle \rightarrow |S\rangle, \quad (1)$$

where  $C_S(\theta_k)$  is a general reversible oracle operation selectively applying to the solution state  $|S\rangle$  to the unstructured search problem. The solution to the search problem can be obtained directly from the solution state  $|S\rangle$ . The solution state also is called the marked state of the search problem. An adiabatic quantum search algorithm also may be efficiently reduced to the unitary quantum circuit (1) [27a]. This quantum search algorithm (1) consists of the two types of unitary operations. One of which is the oracle operations  $\{C_S(\theta_k)\}$ , here each oracle operation may be implemented by the reversible operation sequence  $C_S(\theta_k) = U_f V(\theta_k) U_f$  consisting of the two oracle functional operations  $U_f$  and a conditional phase shift operation  $V(\theta_k)$  [21a]. Another consists of the known unitary operations  $\{U_k\}$ . The reversible oracle operations  $\{C_S(\theta_k)\}$  in (1) are independent on any symmetric structure and property of the Hilbert space that serves as the search space. The known unitary operations  $\{U_k\}$  could be constructed according to the symmetric structure of the Hilbert space, but they also may be built up without considering the symmetric structure of the Hilbert space [39, 24]. Therefore, it may be thought that the quantum search algorithm is independent on any symmetric structure and property of the Hilbert space. Then this means that the search space of the quantum search algorithm is not essentially different from the unstructured search space of the search problem, even though it is just taken as the tensor-product Hilbert space of the  $n$ -qubit spin system. The final state  $|\Psi_f^S\rangle$  in (1) is required to be close

to the solution state  $|S\rangle$  or to contain the solution state in a high probability close to 100%, so that the solution state can be found by quantum measuring the final state  $|\Psi_f^S\rangle$ . This directly leads to that number of the oracle operations in (1) (or equivalently the oracle functional operations) needs to take  $O(\sqrt{N})$  [23, 24]. On the other hand, a classical search algorithm needs to evaluate the oracle function  $O(N)$  times to determine the solution  $S$  that corresponds to the solution state  $|S\rangle$  in a quantum search algorithm. Therefore, in comparison with the classical search algorithm the quantum search algorithm (1) achieves only a square speedup. This is the square speedup limitation on a conventional quantum search algorithm. It has been shown that the quantum search algorithm (1) obeys generally the square speedup limitation [26, 27], here the oracle operation in (1) may be taken as a general one  $C_S(\theta_k)$  with variable angle  $\theta_k$  or the special one  $C_S(\pi)$  with the fixed angle  $\pi$ .

In a polynomial-time quantum search process the efficient reduction from the exponentially large search space to some polynomially small subspace of the Hilbert space is carried out by the unitary quantum circuit:

$$|\Psi_f\rangle = U_k C_S(\theta_k) U_{k-1} C_S(\theta_{k-1}) \dots U_1 C_S(\theta_1) |\Psi_0\rangle \rightarrow \sum_{L \neq S} B_L^S |L\rangle. \quad (2)$$

This quantum circuit is completely the same as that one of (1) in form. However, in the final superposition state  $|\Psi_f\rangle$  of the quantum circuit (2) there is not the solution state  $|S\rangle$  or the solution state  $|S\rangle$  can be neglected with respect to the other states. This is essentially different from the usual quantum search algorithm (1). This is also a necessary condition to realize a reduction from the exponentially large search space to a polynomially small one. Only when this necessary condition is met, can the exponentially large search space be possibly reduced to a polynomially small subspace. In fact, if there is the solution state  $|S\rangle$  in the final state  $|\Psi_f\rangle$ , then the search space is always exponentially large, because the solution state  $|S\rangle$  can be any quantum state of the exponentially large search space. Because the amplitude of the solution state  $|S\rangle$  is zero or negligible in the final state  $|\Psi_f\rangle$ , there is not the square speedup limitation on the quantum circuit (2) that is used to realize the search-space reduction. Therefore, the quantum circuit (2) could be realized efficiently. Because there is not the solution state  $|S\rangle$  in the final state, the original search space of the unstructured search problem and its symmetric structure disappear. This is a necessary condition for a quantum search process to be able to solve an unstructured search problem in polynomial time. The usual quantum search algorithm (1) still works in the original unstructured search space. It is not able to solve an unstructured search problem in polynomial time. Now one may ask whether or not the information of the solution state  $|S\rangle$  also disappears. Actually, the information of the solution state  $|S\rangle$  is transferred to those quantum states  $\{|L\rangle\}$  with  $|L\rangle \neq |S\rangle$  (or their amplitudes  $\{B_L^S\}$ ) of the Hilbert space of the quantum system that serves as the search space. It is clear that the final state  $|\Psi_f\rangle$  does not carry the information of the solution state as a whole but the information of the component states of the solution state. By comparing the two quantum

circuits (1) and (2) with each other one can see that there exist the two extreme cases. One extreme case is that the final state  $|\Psi_f^S\rangle$  in (1) is completely the solution state  $|S\rangle$  for a usual quantum search algorithm. Another is that the final state  $|\Psi_f\rangle$  in (2) does not contain the solution state  $|S\rangle$  at all for the search-space reduction in a polynomial-time quantum search process. This result is very surprising. For a long time researchers have made continuously a great effort to amplify maximally the amplitude of the solution state  $|S\rangle$  with the minimum number of the oracle operations in a quantum search algorithm. However, the successful direction to solve efficiently the unstructured search problem may be the opposite direction to the amplitude amplification of the solution state.

In a polynomial-time quantum search process the original unstructured search space and its symmetric structure are first eliminated so that they cannot affect effectively the quantum search process. At the same time the information of the component states of the solution state is transferred to the tensor-product Hilbert space of the quantum system. Here the tensor-product symmetric structure of the Hilbert space is of crucial importance, because without the tensor-product symmetric structure the information of the component states will not be smoothly transferred into the Hilbert space. Because the information treatment for the component states is the main task in a polynomial-time quantum search process, the unitary operations, excitations, and processes in the component systems of the quantum system are more important in the quantum search process. This could be different from a usual quantum search algorithm, where the information treatment for a quantum state as a whole is the main task. Unitarily manipulating the oracle operations (or the oracle functional operations) [21, 29] is necessary to realize the transfer of information of the component states of the solution state from the original search space into the Hilbert space of the quantum system. This is essentially different from a usual quantum search algorithm [23, 24, 25]. Unitarily manipulating a functional operation which is not an oracle operation is also very important in quantum computing [40]. It is generally required in quantum computation that unitary manipulation and control obey the mathematical logic principles of a problem such as the unstructured search problem. This is different from conventional unitary manipulation and control in quantum mechanics. The symmetric structure and property of the Hilbert space must be considered explicitly in the unitary manipulation in a polynomial-time quantum search process, resulting in that the unitary manipulation is performed mainly in the component systems of the quantum system. Therefore, although both the quantum circuits (1) and (2) are the same in form, they are essentially different from each other. On the one hand, because the mathematical logic principles have to be obeyed, both the quantum circuits (1) and (2) must contain the same reversible oracle functional operations. On the other hand, the known unitary operations  $\{U_k\}$  in (2) which are purely quantum-mechanical can not be taken as arbitrary unitary operators, because they are used for the purposeful unitary manipulation on the oracle operations, and thus, they are unlike those known unitary operations in (1) which are also purely quantum-mechanical. It is this purposeful unitary manipulation that is

used to suppress or even cancel the effect of the original search space and its symmetric structure on the quantum search process and at the same time realize the transfer of information of the component states. *Here the mutual cooperation between the reversible oracle operations and the known unitary operations in the frame of the unitary quantum dynamics reflects the importance of cooperation between the mathematical logic principles of a problem to be solved and the quantum-mechanical principles in quantum computation.*

Though the original search space and its symmetric structure disappear and there does not exist the square speedup limitation on the quantum circuit (2) of the search-space reduction, these could not guarantee that the new search space, i.e., the Hilbert space is not exponentially large for the unstructured search problem. For example, the solution state could disappear apparently, and it could be changed merely from one form to another. At present it is unclear in what conditions the Hilbert space is not exponentially large for the unstructured search problem. However, it is essential for the transfer of information of the component states of the solution state from the original search space into the Hilbert space. This is because the Hilbert space has a tensor-product symmetric structure. The symmetric structure and property of the Hilbert space could be exploited to suppress or even cancel the effect of the symmetric structure of the original search space on the quantum search process. *More importantly one may make use of the symmetric structure and property of the tensor-product Hilbert space to help solving the unstructured search problem* [21, 37, 29]. Information of all these component states of the solution state contains the information of the solution state as a whole. If the information of all these component states is extracted from the Hilbert space, then the solution state can be determined completely. Therefore, extracting the information of all these component states is really equivalent to solving the unstructured search problem. Around ten years ago [21] the present author attempted to extract directly the information of these component states from the Hilbert space of an  $n$ -qubit spin system. The problem for the scheme is that either the information transfer is inefficient (the main one) or the information of all these component states is distributed in the whole Hilbert space which is exponentially large, resulting in that it is hard to extract the information or it needs to take an exponential time to extract the information. Thus, this scheme may be useful only to solve a small-scale search problem [21]. The key strategy to solve the problem is to make use of the symmetric structure and property of the Hilbert space to help extraction of the information of these component states [21, 37, 29]. According to this strategy a crucial method to solve the problem is with the aid of the symmetric structure and property of the multiple-quantum operator spaces of a spin system [21a] which also is a Hilbert operator space. With the help of the symmetric structure and property the information of the component states of the solution state is dynamically transferred to a small state subspace (or state subset) or even a polynomially small state subspace (or state subset) of the Hilbert space, and at the same time the solution state disappears. Here the polynomially small subspace stores only the information of the component states of the solution state and it does not contain the information of the solution state as a whole.

After this information transfer it may be thought that the original unstructured search space is reduced to the polynomially small subspace of the Hilbert space. Dynamically this reduction may be efficiently realized by unitarily manipulating the oracle operations in the component systems of the quantum system [21, 29]. Here emphasize that dynamically the unitary manipulation must be explicitly applied to the component states of the solution state, so that the information transfer can be realized at the same time. This efficient reduction is one of the two key steps to solve the unstructured search problem in polynomial time. It simplifies greatly the extraction of the information of the component states. However, it is still hard to solve the unstructured search problem. This is because amplitude of the quantum state carrying the information of the component states is exponentially small in the polynomially small subspace, resulting in that extraction of the information is still hard. The amplitude is even exponentially smaller than the counterpart in a usual quantum search algorithm. This could be the price for the efficient reduction from the original unstructured search space to the polynomially small subspace of the Hilbert space. In order to extract efficiently the information of the component states in the polynomially small subspace one needs to use the exponential quantum-state-difference amplification [30, 56], which the present author spends the last five years to set up. This exponential quantum-state-difference amplification is extremely powerful. It is another key step to solve the unstructured search problem in polynomial time.

As stated before, there are two fundamental quantum-mechanical principles to help an unstructured quantum search process to bypass the square speedup limitation. One of which is the symmetric structure and property of the tensor-product Hilbert space of a quantum system. It has been discussed in detail above. Another is the unitary quantum dynamics in time and space. A quantum system like a spin system is simpler to realize a quantum computation. Its time evolution process may be described simply by a space-independent unitary quantum dynamics. Today most efficient quantum algorithms are constructed based on the quantum system. From the point of view of unitary manipulation such a quantum system has only one manipulating freedom degree of the internal motion of the system. On the other hand, a quantum system such as a single atom motioning in time and space has not only the internal (electron or spin) motion but also the center-of-mass motion in space. Its time evolution process has to be described by a time- and space-dependent unitary quantum dynamics. Both the polynomial-time unstructured quantum search processes [56] and the reversible and unitary halting protocol [30] are realized in a time- and space-dependent quantum system. Such a quantum system is far more complicated than a spin system, but it may be much more useful in the unitary manipulation to realize the unitary dynamical state-locking process [30, 41, 42, 43] and the exponential quantum-state-difference amplification [30, 56]. From the point of view of unitary manipulation a time- and space-dependent quantum system may have two or more independent manipulating freedom degrees. In particular, a single atom motioning in time and space is one of the simplest time- and space-dependent quantum systems. It has two independent manipu-

lating freedom degrees. One of which is the atomic internal motion and another the atomic center-of-mass motion. A single atom that has both the independent manipulating freedom degrees may be used to realize the quantum-state-level mutual cooperation between both the atomic internal and center-of-mass motions. Here realization for the quantum-state-level mutual cooperation needs to manipulate unitarily the discrete atomic internal motion, the atomic center-of-mass motion, and the coupling between the atomic internal and center-of-mass motions [30, 41, 42, 43]. *Such quantum-state-level mutual cooperation plays the key role in realizing the unitary dynamical state-locking process* [30]. In general, a unitary dynamical state-locking process may be defined simply and intuitively as a unitary process that transforms simultaneously two (or more) very distinguishable quantum states (e.g., a pair of orthogonal states) to two (or more) indistinguishable quantum states (e.g., a pair of non-orthogonal states) whose difference may be arbitrarily small. A unitary dynamical state-locking process may be used to realize the reversible and unitary halting protocol [30]. The essential difference between a unitary dynamical state-locking process and a conventional unitary process (or operation) may be seen intuitively through the following typical instance. Consider that a single atom in a harmonic potential field is in the product state  $|s_k\rangle|\Psi(x, t_0)\rangle$ . Here the discrete atomic internal state  $|s_k\rangle$  may be either  $|0\rangle$  or  $|1\rangle$  and the atomic center-of-mass motional state  $|\Psi(x, t_0)\rangle$  may be the ground state of a harmonic oscillator, which is a Gaussian wave-packet state. Now the two orthogonal internal states  $|0\rangle$  and  $|1\rangle$  may be transformed to the desired internal state  $|0\rangle$  simultaneously in a probability close to unity by the unitary dynamical state-locking process  $U_{DSL}$  in this single-atomic system [30, 56],

$$U_{DSL}|s_k\rangle|\Psi(x, t_0)\rangle \rightarrow \{\rho(s_k)|0\rangle + \exp[i\gamma(s_k)]\sqrt{1 - |\rho(s_k)|^2}|1\rangle\}|\Psi(x, t_0)\rangle, \quad (3)$$

where  $|\rho(s_k)|^2$  is the probability that the internal state  $|s_k\rangle$  with  $s_k = 0$  or  $1$  is changed to the desired state  $|0\rangle$  and  $\exp[i\gamma(s_k)]$  is a phase factor. The unitary dynamical state-locking process (3) may be realized efficiently. The minimum one of the two different probability values  $\{|\rho(s_k)|^2\}$  in (3) is defined as  $P_{\min} = \min\{|\rho(0)|^2, |\rho(1)|^2\}$ . The unitary dynamical state-locking process is essentially different from a usual unitary process (or operation) in that the minimum probability  $P_{\min}$  ( $P_{\min} < 1$ ) can be made infinitely close to unity without destroying the unitarity of the process [30, 56]! This extremely important property leads to that a unitary dynamical state-locking process and its inverse process together can realize an exponential quantum-state-difference amplification and hence an efficient quantum search process becomes possible.

A quantum-state-difference amplification is just the inverse process of a unitary dynamical state-locking process such as (3). An exponential quantum-state-difference amplification means that the inverse process of a unitary dynamical state-locking process such as (3) can be realized in polynomial time even when the difference between both the non-orthogonal states on the right-hand side of (3) is exponentially small. As a typical example, on the right-hand side of (3) both the non-orthogonal states  $|0\rangle$  and  $A(|0\rangle + 2^{-n}|1\rangle)$  (the normalization



constant  $A \approx 1$  for the qubit number  $n \gg 1$ ) whose difference is exponentially small may be transformed simultaneously to the two orthogonal states  $|0\rangle$  and  $|1\rangle$  by the exponential quantum-state-difference amplification of (3) in polynomial time, respectively. This illustrates that an exponential quantum-state-difference amplification is extremely powerful to distinguish two non-orthogonal states unambiguously. Can an exponential quantum-state-difference amplification help to solve efficiently an unstructured search problem? Consider the two non-orthogonal states  $|\Psi_k^S\rangle$  and  $|\Psi_k^0\rangle$  [26b, 26c] that are created via the quantum circuit (1) of a usual quantum search algorithm and its modified version without containing any oracle operations, respectively,

$$|\Psi_k^S\rangle = U_k C_S(\theta_k) U_{k-1} C_S(\theta_{k-1}) \dots U_1 C_S(\theta_1) |\Psi_0\rangle,$$

$$|\Psi_k^0\rangle = U_k U_{k-1} \dots U_1 |\Psi_0\rangle,$$

here the oracle number  $k = \text{poly}(n)$ . Clearly only the state  $|\Psi_k^S\rangle$  contains the information of the solution state  $|S\rangle$ . In most cases both the known state  $|\Psi_k^0\rangle$  and the unknown state  $|\Psi_k^S\rangle$  have an exponentially small difference [26]. If both the non-orthogonal states  $|\Psi_k^0\rangle$  and  $|\Psi_k^S\rangle$  could be transformed to their corresponding orthogonal states by an exponential quantum-state-difference amplification in polynomial time, then one would be able to distinguish unambiguously the state  $|\Psi_k^S\rangle$  from the state  $|\Psi_k^0\rangle$ , leading to that the solution state  $|S\rangle$  could be found and the unstructured search problem could be solved efficiently. However, as stated below, there is a constraint on the power of an exponential quantum-state-difference amplification. In fact, a unitary dynamical state-locking process can be realized in polynomial time only for polynomially many quantum states of the Hilbert space. It is not able to transform simultaneously all the quantum states of the exponentially large Hilbert space into their corresponding non-orthogonal states whose differences can be made exponentially small in polynomial time. This directly leads to that an exponential quantum-state-difference amplification is not capable of distinguishing all the quantum states from one another in the exponentially large Hilbert space. This constraint on the power of an exponential quantum-state-difference amplification results in that one is not able to use an exponential quantum-state-difference amplification to distinguish unambiguously the unknown state  $|\Psi_k^S\rangle$  from the known state  $|\Psi_k^0\rangle$ , because the unknown state  $|\Psi_k^S\rangle$  is of the exponentially large search space. This leads to that the unstructured search problem can not be solved efficiently, and this result does not destroy the square speedup limitation. Whether or not the exponentially large search space can be reduced to a polynomially small state subspace (or subset) in polynomial time therefore is essential for a polynomial-time unstructured quantum search process, because only when this efficient search-space reduction can be realized via the quantum circuit (2), can an exponential quantum-state-difference amplification be possibly available for exponentially speeding up the quantum search process.

The unitary quantum dynamics has the unique ability to build the effective interaction between the symmetric structure and property of the tensor-product Hilbert space of a quantum system (or the quantum mechanical principles) and

the mathematical symmetric structure of a problem to be solved on the quantum system (or the mathematical logic principles). It is this effective interaction that can be used to realize efficiently the information transfer from one form, i.e., the information of the solution state as a whole to another, i.e., the information of the component states of the solution state, and from the exponentially large search space of the unstructured search problem to a polynomially small subspace (or subset) of the tensor-product Hilbert space of the quantum system. Thus, the effective interaction is essentially important to realize a polynomial-time quantum search process. This fact reflects the inherent importance for the unitary quantum dynamics to speed up a quantum computation. On the other hand, an exponential quantum-state difference amplification, another part of the polynomial-time quantum search process, is also constructed based on the unitary quantum dynamics in time and space. Therefore, both the unitary quantum dynamics and the symmetric structure and property of the tensor-product Hilbert space of a quantum system are responsible for the exponential speedup of a polynomial-time quantum search process. This mechanism for exponential quantum-searching speedup also is very important to understand the essence of exponential speedup of other efficient quantum algorithms.

### 3. The unitary quantum dynamics and the universal quantum driving force to speed up a quantum computation

The unitary quantum simulation of nuclear spin systems or ensembles has been studied extensively by the present author [36, 35, 37], which is related to the author's research on the nuclear spin dynamics in the early 1990s. The unitary quantum dynamics has guided the author to construct the quantum search processes in the past decade, although the constructed quantum search process evolves from one form to another [21, 37, 29, 30]. It also leads the author to discover the polynomial-time quantum search processes [56]. As described above, it plays the central role in constructing a polynomial-time quantum search process. Therefore, the unitary quantum dynamics is closely related to a quantum-computing speedup. However, the inherent importance for the unitary quantum dynamics to speed up a quantum computation was not revealed until these extraordinarily fast quantum algorithms [21, 32] are discovered by the present author in the early 2000s (See also the comment [33]). Since then, the unitary quantum dynamics including the time- and space-dependent one has been considered by the author as the fundamental and universal principle to conduct research, construction, and realization of a quantum computational process [21, 32, 40, 37, 29, 30, 41, 42, 43], although it has been suspected extensively that the unitary quantum dynamics is a fundamental principle in macroscopic world. The principle states explicitly that *both a closed quantum system and its quantum ensemble obey the same unitary quantum dynamics*. Here the closed quantum system is referred to a pure-state system whose density operator  $\rho$  satisfies  $Tr(\rho^2) = 1$ , while its quantum ensemble satisfies  $Tr(\rho^2) < 1$ . It should be pointed out that a polynomial-time quantum search process [56] is constructed

in a pure-state quantum system and it has nothing to do with a quantum ensemble. But according to this fundamental and universal principle the mechanism for exponential quantum-searching speedup could be generally applicable not only in a pure-state quantum system but also in a quantum ensemble. In turn, if this exponential speedup mechanism is generally available [21, 32, 40, 33, 56] (See also the comment [48]), then this could indicate strongly that the unitary quantum dynamics is the fundamental and universal principle not only in a closed pure-state quantum system but also in a closed quantum ensemble although it has been suspected extensively that the unitary quantum dynamics is a fundamental principle in a macroscopic physical system [47]. This problem will be further discussed later. Below it will be shown that the unitary quantum dynamics in quantum mechanics is the universal quantum driving force to speed up a quantum computation.

A universal quantum computational model [4, 5, 6, 7] obeys the three fundamental principles (or properties): *(i)* the unitary quantum dynamics, *(ii)* the mathematical-logic principles, and *(iii)* the quantum-state description associated with a variety of fundamental quantum-state properties. According to quantum mechanics [9] the first principle *(i)* is naturally compatible with the third one *(iii)* in a quantum system to execute a quantum computation. On the other hand, it may be considered that the first principle *(i)* is compatible with the second principle *(ii)* in the sense that every reversible mathematical-logic operation in a quantum computation may be replaced with a discrete unitary operation. This consideration is based on both the reversible classical computational models [2] and the quantum Turing machine models [3]. However, there is a problem that the second principle *(ii)* may not be always compatible with the third one *(iii)* in a quantum computation in a universal quantum computational model. The mathematical logic principles of a problem to be solved put a constraint on the unitary quantum dynamics and also the possible application of the quantum-state properties in a quantum computation. If in a quantum computation one makes use of the quantum-state properties of a quantum system which may be a pure-state system or a mixed-state ensemble, then there could meet a conflict between the mathematical logic principles and the quantum-state properties of the quantum system in the quantum computation. In fact, such an inherent conflict is quite general in a quantum computational process that needs to perform many different reversible (mathematical-logic) functional operations in the Hilbert space of a quantum system with a fixed number of qubits. For example, the unitary manipulation on functional operations in a quantum computational process needs to perform many reversible functional operations. Note that this conflict is independent of any quantum measurement. Hence it is within the universal quantum computational models under study at present. It is different from the inherent conflicts early found [44, 45] due to the quantum measurement in the halting operation in the universal quantum Turing machine model [4]. However, all these conflicts inherent in the universal quantum computational models can not lead to that one of the three fundamental principles as stated above is more essential than the two other principles in a quantum computation and especially in speeding up a quantum

computation. In particular, from these conflicts one is not able to deduce that the unitary quantum dynamics is in the priority position in a universal quantum computational model with respect to the mathematical-logic principles and the fundamental quantum-state properties. *Actually, a universal quantum computational model itself is not able to decide which one of these fundamental principles is more essential than the others in a quantum computation. It is not yet able to resolve its inherent conflict problems by itself.* Resolving these problems has to be based on the quantum-computing speedup theory.

Here, using the quantum measurement to do a quantum computation is considered to be beyond the universal quantum computational models under study at present. It is not discussed in the paper. A universal quantum computational model should be inherently consistent. One of the reasons why here the quantum measurement is not included in these universal quantum computational models is that too many conflicts will be generated among these fundamental principles if the quantum measurement is added to these universal quantum computational models. As an example, in the universal quantum Turing machine model [4] both the unitary computational process and the quantum parallel principle tend to conflict with the halting operation [44, 30, 45] which is really involved in the quantum measurement. For simplicity, the relativistic effect and the decoherence effect of a quantum system are not yet considered in the paper.

A quantum-computing speedup theory studies mainly the mechanisms of quantum-computing speedup and especially exponential quantum-computing speedup. A quantum-computing speedup is used to measure the computational power difference between quantum computation and classical computation when both a quantum algorithm and the best classical algorithm solve a same computational problem. A reasonable quantum-computing speedup theory should satisfy some necessary conditions including: (a) It is able to provide reasonable mechanisms of exponential quantum-computing speedup for the discovered polynomial-time quantum algorithms; (b) It may lead to that a universal quantum computational model is inherently consistent; (c) It should respect the fundamental and universal principle, i.e., the unitary quantum dynamics; (d) It is able to guide construction and realization of a quantum algorithm and especially an efficient quantum algorithm. Here it is required that the condition (c) be satisfied mainly because the unitary quantum dynamics plays the central role in constructing a polynomial-time quantum search process. It also seems reasonable to require that the quantum-computing speedup theory be consistent with the quantum parallel principle in explaining the exponential speedup for the existing efficient quantum algorithms. The conditions (a) and (c) are in the priority position because a quantum-computing speedup theory studies mainly the mechanisms of quantum-computing speedup. It seems reasonable and natural that *in the quantum-computing speedup theory the unitary quantum dynamics is positioned at the center of a universal quantum computational model.* However, it is impossible to show that the unitary quantum dynamics is superior to the two other fundamental principles (or properties) within a universal quantum computational model. The unitarity of the physical process of a quantum computation is a necessary but not sufficient condition to set up a universal

quantum computational model, because unitarity of a pure quantum physical process does not ensure that the mathematical-logic principles are naturally satisfied in the process. The mathematical-logic reversibility is also a necessary but not sufficient condition to set up a universal quantum computational model, because it can only ensure the reversibility of a computation in mathematical logic but not the unitarity of the physical process of the computation. Therefore, a microscopic physical process may obey the mathematical-logic principles or may not, although it obeys the unitary quantum dynamics. From the point of view of computational performance it seems that the quantum-state properties are even more important for a quantum computation to outperform a classical computation, and this has been supported by the investigation on the computational performance of a universal quantum computational model in the past two decades.

There are several important reasons why the unitary quantum dynamics should be put at the heart of a universal quantum computational model in the quantum-computing speedup theory. First of all, it is clear that such a theory respects the fundamental and universal principle, i.e., the unitary quantum dynamics and puts the principle in the priority position in a universal quantum computational model. Next, if one puts the unitary quantum dynamics at the heart of a universal quantum computational model, then the conflict between the second (*ii*) and third (*iii*) principles as stated above may be avoided. Obviously, there is not such a conflict for a pure unitary quantum simulation. The conflict is met only when a quantum computation is carried out. Now suppose that a computational problem is solved by a quantum computational process on a quantum computer. Then the mathematical logic principles that the problem obeys must be carried out in a reversible or unitary form in the quantum computational process. Since the unitary quantum dynamics is in the priority position, the quantum-state properties that can be used in the computational process have to be suitably constrained so that both the unitary quantum dynamics and the mathematical logic principles are satisfied at the same time in the computational process. This really means that there is not any conflict in the computational process. Therefore, the quantum-computing speedup theory immediately results in a fundamental rule to guide construction and realization of a quantum algorithm in a universal quantum computational model that a quantum computational process obeys the unitary quantum dynamics and is compatible with the mathematical logic principles in a reversible or unitary form. This fundamental rule has been used by the present author to guide research, construction, and realization of a quantum computational process in the past years [21a, 29]. It is also the basic starting point of the present work. Another particularly important reason is discussed below that is relevant to the realization of mathematical logic principles in quantum computation.

A computation is to use a physical device (or computer) to treat a computational task (or problem) according to a specific set of fundamental mathematical-logic operation rules. In the computation the mathematical logic principles that the problem obeys also are carried out by a specific set of fundamental mathematical-logic operation rules. In the classical Turing machine models [31]

these fundamental mathematical-logic operation rules are realized in irreversible form, while in the reversible classical computational models [2] these fundamental operation rules are implemented in reversible form. In these classical computational models these fundamental operation rules have to be obeyed strictly during a computation and hence these fundamental mathematical-logic operations are not allowed to be manipulated. In the quantum Turing machine models [3] the fundamental mathematical-logic operation rules are realized in unitary form. Just like the reversible classical computational models, the quantum Turing machine models do not allow these fundamental mathematical-logic operations to be manipulated. Similarly, in the universal quantum computational models [4, 5, 6, 7] it is still required that these fundamental mathematical-logic operation rules be strictly performed in reversible or unitary form. *In fact, there is the most basic requirement for any computational model that fundamental mathematical-logic operation rules be strictly and correctly carried out in a computation.* This directly leads to that the mathematical logic principles of a problem to be solved have to be satisfied in a reversible or unitary form in a quantum computation in a universal quantum computational model [21a, 29]. A universal quantum computational model usually does not yet allow these fundamental mathematical-logic operations to be manipulated. However, there is something more than the fundamental mathematical-logic operation rules in a general quantum computational process. There could exist a possibility to manipulate unitarily the mathematical-logic operations of a problem to be solved in a universal quantum computational model if purely quantum-mechanical operations, excitations, and processes and so on are allowed to use in the universal quantum computational model and the unitary manipulation does not destroy the mathematical-logic principles that the problem obeys. Here these purely quantum-mechanical operations, excitations, or processes need not obey the fundamental mathematical-logic operation rules, but they obey merely the quantum-mechanical principles. The quantum-computing speedup theory also studies how these purely quantum-mechanical operations, excitations, or processes manipulate the mathematical-logic operations to speed up a quantum computation. It bridges the gap between quantum computation and purely quantum-mechanical quantum simulation.

In general, mathematical-logic principles including mathematical symmetrical structures of a problem to be solved do not affect essentially computational performance, because they are independent on a detailed physical computational model. A similar viewpoint also may be seen in Refs. [4, 5]. More intuitively, a computational problem is independent of a computer. Thus, it is impossible that the mathematical-logic principles that the problem obeys are able to affect essentially computational performance of the computer. Though these mathematical logic principles can not affect essentially the computational performance, the computational process to solve the problem has to obey these mathematical logic principles [21a] no matter whether the computer is classical or quantum. Thus, a quantum-computing speedup to solve a problem makes sense only when the mathematical-logic principles of the problem are first obeyed in a quantum computational process. Of course, the mathematical-

logic principles also are obeyed in a classical computational process to solve the same problem. However, here emphasizes the quantum computational process because a quantum-computing speedup is determined alone by the quantum physical laws. Why a quantum-computing speedup is determined only by the quantum physical laws? Computational powers for the classical and quantum computers are determined only by the classical and quantum physical laws, respectively. Since the computational power for the best classical computational process is unique, the quantum-computing speedup which is used to measure the computational-power difference between quantum computation and classical computation is only dependent on computational power of the quantum computational process and hence it is determined alone by the quantum physical laws. The precondition that a quantum-computing speedup makes sense tends to be neglected in quantum computation. Researchers do not pay attention to this precondition possibly because that fundamental mathematical-logic operation rules are strictly and correctly carried out in a computation is the most basic requirement for any computational model. This precondition shows that whether or not a fundamental quantum physical attribute (e.g., the unitary quantum dynamics, quantum superposition principle, quantum coherence interference, and so on) is a universal quantum driving force to speed up a quantum computation depends on whether or not this fundamental attribute is able to perform independently and correctly mathematical-logic operations in computation. Though the mathematical logic principles do not affect essentially quantum computational performance, they can determine which fundamental attribute in quantum mechanics may not be a universal quantum driving force to speed up a quantum computation. On the other hand, the polynomial-time quantum search processes [56] tell ones that the effective interaction between the mathematical-logic principles and the quantum physical laws plays a crucial role in achieving an essential speedup for a quantum search process. This shows that only those fundamental attributes in quantum mechanics that are able to build the effective interaction may be considered as candidates of universal quantum driving force to speed up a quantum computation. A fundamental quantum-physical attribute may be a universal quantum driving force only if it can perform independently and correctly mathematical logic operations in a quantum computation and it is able to build the effective interaction between the mathematical logic principles and the quantum physical laws. This is the necessary condition that a fundamental attribute is able to become a universal quantum driving force to speed up a quantum computation. It is well known that these fundamental quantum-physical attributes including quantum superposition principle, quantum coherence interference, quantum-mechanical symmetry, the properties of quantum entanglement states, the properties of multiple-quantum coherence mixed states, quantum measurement, and so on, can not perform independently and correctly mathematical logic operations in computation. Thus, all these fundamental attributes are not a universal quantum driving force to speed up a quantum computation. It is also impossible for the decoherence effect and nonlinear effect (if it existed in a quantum system) to be a universal quantum driving force to speed up a quantum computation.

Note that the reversible classical mechanics and equilibrium-state thermodynamics could perform independently the mathematical logic operations. But because they can not build the effective interaction between the mathematical logic principles of a problem to be solved and the symmetric structure and property of the Hilbert space, they are not a universal quantum driving force to speed up a quantum computation. It is well known that the mathematical-logic principles can be carried out independently and correctly by the unitary quantum dynamics. On the other hand, the polynomial-time quantum search processes [56] tell ones that the unitary quantum dynamics has the unique ability to build the effective interaction between the mathematical logic principles and the quantum physical laws. *Therefore, the unitary quantum dynamics in quantum mechanics is the universal quantum driving force to speed up a quantum computation.* When people are talking about a quantum-computing speedup in a quantum computation, they can not ignore the unitary quantum dynamics that is restricted by the mathematical logic principles. Otherwise, their obtained conclusion could make no sense! A typical example may be seen in the comment [52]. These facts strongly support that the unitary quantum dynamics has to be put at the center of a universal quantum computational model.

It is very important that the unitary quantum dynamics in quantum mechanics is the universal quantum driving force to speed up a quantum computation. This result supports strongly that the mechanism for exponential speedup deduced from the polynomial-time quantum search processes could be generally available not only in a pure-state quantum system but also in a quantum ensemble. In turn, this further implies that the unitary quantum dynamics is the fundamental and universal principle not only in a pure-state quantum system but also in a quantum ensemble, although it has been suspected extensively that the unitary quantum dynamics is a fundamental principle in a macroscopic physical system. Therefore, the quantum-computing speedup theory supports strongly in theory that the unitary quantum dynamics is the fundamental and universal principle in nature.

In the past two decades there are a large number of works to apply simply the unitary time evolution processes (here relaxation or decoherence is inevitable) of a physical system which may consist of spins, atoms, molecules, or electrons, to implementing experimentally the quantum logic gates, simple quantum algorithms, quantum simulations, and so on. These works merely make use of the unitary quantum dynamics. They have little help for studying and understanding the essence of the unitary quantum dynamics to speed up a quantum computation. Understanding the unitary quantum dynamics based on these works is not deeper and not yet more essential than the one based on the conventional nuclear magnetic resonance (NMR) experiments and electronic spin resonance (ESR) experiments (See also the comment [33]). Therefore, these works have little help for understanding the essential reason why the unitary quantum dynamics is considered as the fundamental and universal principle in nature. For example, a protein-folding process is a typical non-equilibrium process in nature. The unitary quantum dynamics is destroyed strongly in the process. How can one say the unitary quantum dynamics is a fundamental principle in such



a process? How can one say the unitary quantum dynamics governs such a process? Only the mechanism for exponential quantum-searching speedup can predict that *such a process that does not obey the unitary quantum dynamics is governed by the unitary quantum dynamics*.

As the central position of a universal quantum computational model the unitary quantum dynamics could help to resolve the inherent conflicts appearing in the universal quantum computational models. The inherent conflicts [44, 30, 45] among the unitary quantum dynamics, the quantum parallel principle, and the halting operation appear in the universal quantum Turing machine model [4] due to the quantum measurement in the halting operation. They include the conflict [44, 30] between the unitary quantum dynamics and the quantum measurement and the conflict [44b, 30] between the quantum parallel principle and the halting operation. There is not the priority position for any one of the unitary quantum dynamics, the quantum parallel principle, and the halting operation. In the paper [30] the present author suggested that these conflict problems could be resolved on the basis of the unitary quantum dynamics. This suggestion is based on the fact that the unitary quantum dynamics is important in speeding up a quantum computation. Now the unitary quantum dynamics is put at the center of the universal quantum Turing machine model owing to its inherent importance to speed up a quantum computation. Then the strategy in Ref. [30] to resolve these inherent conflict problems becomes reasonable. A similar strategy also may be adopted to resolve other conflict problems than those in Refs. [44, 30] in a universal quantum computational model, as described above. Actually, a universal quantum computational model itself can not resolve its inherent conflict problems. Resolving these inherent conflict problems must be based on the quantum-computing speedup theory.

More importantly, the unitary quantum dynamics provides the fundamental frame to realize the effective interaction between the mathematical-logic principles of a problem to be solved and the quantum physical laws. A universal quantum computational model also obeys the quantum-mechanical symmetry. Quantum-mechanical symmetry of a quantum system that is different from both the fundamental quantum-state properties and the unitary quantum dynamics is a fundamental attribute of quantum world. The quantum-mechanical symmetry under consideration here is mainly referred to the symmetric structure and property of the Hilbert space of a composite quantum system. It is based on the basic postulate in quantum mechanics that *the whole Hilbert space of a composite quantum system is a direct product (or tensor product) of the Hilbert spaces of component systems of the composite system*. The quantum-mechanical symmetry includes the symmetric structure and property of the multiple-quantum operator algebra spaces [35, 21a]. Of course, there are also a number of the traditional quantum-mechanical methods, i.e., the group-theory-based methods [9] (quantum-mechanical symmetry is closely related to group theory) to characterize the symmetric structure of the Hilbert space of a quantum system. But so far it has not yet been shown that these traditional methods are useful to find an efficient quantum algorithm in the frame of the quantum-computing speedup theory. Here stress the multiple-quantum operator spaces because the

multiple-quantum transition processes are the basic quantum physical processes in the quantum systems including multiple-spin systems. Global symmetries of a closed quantum system such as the space translational symmetry, rotational symmetry, and time displacement symmetry, which result in the momentum, angular momentum, and energy conservation laws, respectively, may not affect essentially quantum computational performance. Of course, it does not rule out the possibility that the local versions of these symmetries affect essentially quantum computational performance. According to the quantum-computing speedup theory a quantum-mechanical symmetry is not a universal quantum driving force to speed up a quantum computation. However, in the frame of the unitary quantum dynamics the symmetric structure and property of the Hilbert space may have an essential effect on quantum computational performance. This is because it can affect directly and most effectively the unitary quantum dynamics. The reason behind it is that the symmetric structure and property of the Hilbert space of a quantum system is closely related to the Hamiltonian of the quantum system. The symmetric structure and property may make a direct impact on the Hamiltonian of the quantum system, leading to that the effect of the symmetric structure and property can be on the whole time evolution process of the quantum system. On the other hand, though the mathematical logic principles of a problem to be solved do not affect essentially quantum computational performance, they can make a strong constraint on the unitary quantum dynamics of the quantum system to solve the problem [21a]. This leads to that the unitary quantum dynamics is able to bring together the mathematical logic principles and the symmetric structure and property of the quantum system and make them interacting with each other. As described above, this interaction is of crucial importance to realize a polynomial-time quantum search process.

The unitary quantum dynamics which acts as the fundamental frame also may build the effective interaction between, on the one hand, the symmetric structure and property of the Hilbert space and the time reversal symmetry of a quantum system, and on the other hand, the mathematical logic principles that a computational problem obeys. This leads to a new strategy to construct an efficient quantum algorithm. A typical application of the strategy may be seen in Refs. [21b, 40]. Here the strategy will not be discussed in detail. It must be pointed out that all these methods or strategies need to use the unitary manipulation on the functional operations of the problem to be solved.

#### 4. Quantum parallel principle and quantum-computing speedup

The old quantum-computing speedup theory is based on the quantum parallel principle [4]. The quantum parallel principle uses the quantum superposition principle in quantum mechanics to speed up a quantum computation. It has an intuitive feature similar to the classical parallel computation. The quantum parallel principle has been considered to power essentially a quantum computation in the past decades. It has been thought that this principle may achieve an exponential quantum-computing speedup in some specific cases [11, 12, 13, 14,

15]. However, its generality is limited greatly. In the past decades the quantum physical essence for how the quantum parallel principle speeds up exponentially some specific quantum computations has been investigated extensively. A vast number of works have been devoted to this research subject. It has been thought extensively that the quantum entanglement states could be responsible for the exponential quantum-computing speedup [49]. However, a number of works [50] show that there do not appear quantum entanglement states in the NMR experimental tests of some quantum algorithms. A number of works also suspect or reject that quantum entanglement states can help a quantum computation to achieve an exponential quantum-computing speedup [51]. Some works show that an exponential quantum-computing speedup could have nothing to do with quantum entanglement states [40]. These works reflect mixed effect of quantum states and their fundamental properties and especially quantum entanglement states, multiple-quantum coherence mixed states, etc., and their fundamental properties on a quantum computation.

There are some obstacles for quantum states of the Hilbert space of a quantum system and their fundamental properties to become a quantum driving force to speed up a quantum computation. According to the quantum-computing speedup theory quantum states and their fundamental properties can not be a universal quantum driving force to speed up a quantum computation, as pointed out in the previous paragraph, because quantum states themselves and their fundamental properties are not able to perform independently and correctly mathematical logic operations in computation. Intuitively speaking, a quantum computational process is a dynamical process, while quantum states and their properties are static and passive and thus they can not power independently a quantum computation. Moreover, quantum states themselves are not able to distinguish a quantum computational process from a purely quantum-mechanical dynamical process. Suppose that a black box performs a unitary dynamical process. This process may be a purely quantum-mechanical dynamical process or a quantum computational process that is constrained by the mathematical-logic principles of a problem. Then input and output quantum states of the black box are not able to determine whether the unitary dynamical process is a purely quantum-mechanical process or it is a quantum computational process (See the comment [52]). Note that the intermediate quantum state in the process is completely determined by the input and output quantum states for a given quantum algorithm. Therefore, it is quite complex for quantum states and their fundamental properties to affect a quantum-computing speedup. They could help the unitary quantum dynamics to speed up a quantum computation. They could not have any contribution to a quantum-computing speedup. Even they could have a negative contribution to the quantum-computing speedup. *If quantum states themselves could speed up independently a quantum computation, then it would be very possible that they sped up a non mathematical logic operation:  $1 + 2 = 5$ .*

Though quantum states and their fundamental properties may not be a quantum driving force to speed up a quantum computation, they could be able to help the unitary quantum dynamics to speed up a quantum computation.

Because quantum states and their fundamental properties are independent of the unitary quantum dynamics, their effect on mathematical-logic functional operations usually is not effective. The effect of the quantum states tends to be limited to the initial time of a functional operation. These are the main reason why it tends to be ineffective for quantum states and their fundamental properties to help the unitary quantum dynamics to achieve an exponential quantum-computing speedup except for some special cases [11, 12, 13, 14, 15, 49c, 49d]. Among these fundamental quantum-state properties the quantum superposition principle and quantum coherence interference could be most useful to help the unitary quantum dynamics to speed up a quantum computation.

In the past decades it has been considered extensively that the quantum parallel principle leads to the exponential quantum-computing speedup for the Abelian hidden-subgroup-problem (HSP) quantum algorithms [11, 12, 13, 14, 15] and some special non-Abelian HSP quantum algorithms [16]. It also has been considered that quantum-computing speedup for the conventional quantum search algorithms [23, 24, 25], quantum simulations [18], and other quantum algorithms [49c, 49d] and so on originates from the quantum parallel principle, simply since the quantum superposition states play the central role or the quantum entanglement states appear in these quantum computational (or simulation) processes [49b, 49d]. However, as shown above, quantum states including quantum superposition states and quantum entanglement states can not be a quantum driving force to speed up a quantum computation. Here the quantum-computing speedup theory could provide possible mechanisms for exponential speedup for these efficient HSP quantum algorithms. More importantly, the theory could be helpful for improving essentially these inefficient non-Abelian HSP quantum algorithms. As a typical example, here gives a possible mechanism for exponential speedup of an efficient HSP quantum algorithm on the basis of the unitary quantum dynamics. An HSP quantum algorithm typically consists of the three parts: the initial superposition state [4, 11, 7], a mathematical-logic functional operation (See, for example, Refs. [13, 14, 15a, 53]), and the quantum Fourier transform [14, 15c]. The basic structural characteristic feature for an efficient HSP quantum algorithm is the mutual cooperation between the last two parts in the frame of the unitary quantum dynamics and in the suitable initial superposition state.

The mathematical symmetric structure of a hidden subgroup (HS) problem constrains and modifies the unitary quantum dynamics of the quantum system used to solve the problem. Then this constrained or modified unitary quantum dynamics is applied to the suitable initial superposition state so that the mathematical symmetric structure is mapped into the Hilbert space of the quantum system. This is the first step of a usual HSP quantum algorithm. This mapped symmetric structure in the Hilbert space may be called the problem symmetric structure. It originates from the mathematical symmetric structure of the HS problem alone, although it is in the Hilbert space. It is different from the symmetrical structure of Hilbert space of the quantum system itself. In Ref. [29] a cyclic group state space has a symmetric structure similar to the current one, but that symmetric structure is referred to that one of the Hilbert space of

the quantum system itself. In that case the search space of the search problem coincides with the cyclic group state space (i.e., the Hilbert space). Now the information of solution of the HS problem is hidden in the problem symmetric structure in the Hilbert space. The second step of the HSP quantum algorithm is to apply a suitable quantum Fourier transform to the quantum system so that the solution information hidden in the problem symmetric structure could be extracted in polynomial time from the Hilbert space by the quantum measurement. It has been considered extensively that the second step is the key step to achieve an exponential speedup for the HSP quantum algorithm. It also has been recognized extensively that the problem symmetric structure in the Hilbert space is key important for an efficient HSP quantum algorithm to achieve an exponential speedup. Because the problem symmetric structure is generated by the modified unitary quantum dynamics, the quantum Fourier transform has to precisely cooperate with the modified unitary quantum dynamics in the frame of the unitary quantum dynamics so that an exponential speedup could be achieved in solving the HS problem. This is just the basic structural characteristic feature for a polynomial-time HSP quantum algorithm. Though sometimes the unitary quantum dynamics of the HSP quantum algorithm could be locally destroyed, for example, some unitary operations in the quantum algorithm could be replaced locally with the quantum measurement processes [15b], this basic structural characteristic feature is not changed essentially and hence the exponential speedup for the quantum algorithm is still retained basically.

Because the problem symmetric structure in the Hilbert space originates from the HS problem alone, it has nothing to do with the initial superposition state. This is the main reason why the initial superposition state does not have an essential effect on the exponential quantum-computing speedup to solve the HS problem. This means that the exponential speedup is essentially related to only the modified unitary quantum dynamics and the quantum Fourier transform, although the initial superposition state is a necessary component in an efficient HSP quantum algorithm.

The effect of the initial superposition state, which is the basic character of the quantum parallel principle, on solving efficiently the HS problem lies in that the superposition state helps the modified unitary quantum dynamics to generate the problem symmetric structure in the Hilbert space in polynomial time. In the old quantum-computing speedup theory the superposition state is considered as the key to generating the problem symmetric structure in polynomial time. But the new quantum-computing speedup theory considers that the superposition state does not have any ability to perform in a parallel form any mathematical-logic functional operation, and only the modified unitary quantum dynamics is able to perform independently and correctly the parallel mathematical-logic functional operation. Therefore, the modified unitary quantum dynamics is really the key to generating the problem symmetric structure in polynomial time, although the initial superposition state is a necessary component in the parallel functional operation.

This kind of exponential speedup to solve the HS problem is not general in

quantum computation. It is quite special because only in some special cases the precise mutual cooperation is easy to realize between the modified unitary quantum dynamics and the quantum Fourier transform in the suitable initial superposition state. It is efficient to realize this precise mutual cooperation for an Abelian HS problem [15]. However, it is generally hard to realize the precise mutual cooperation for a non-Abelian HS problem [16] except for some special cases. Different HS problem leads to different modified unitary quantum dynamics. This further leads to that the precise mutual cooperation needs to be re-realized so that an exponential speedup could be re-achieved. If this mutual cooperation could no longer exist due to that the HS problem is changed, then this exponential speedup could disappear.

How can the quantum-computing speedup theory help to solve efficiently a non-Abelian HS problem? Obviously, the quantum algorithm to solve the HS problem does not consider explicitly the symmetrical structure and property of the Hilbert space of the quantum system. This results in that the latter does not have any effective effect on the problem symmetric structure in the Hilbert space. From this point one may say that a usual HSP quantum algorithm is not full quantum. This could be the main reason why this type of efficient quantum algorithms are quite special. Therefore, there could be a possible strategy [40, 21b] to improve these inefficient non-Abelian HSP quantum algorithms that these inefficient quantum algorithms could be improved essentially by making use of the symmetrical structure and property of the Hilbert space of the quantum system. This strategy could be realized only with the help of the unitary manipulation on the mathematical-logic functional operations.

## 5. Discussion and conclusion

In the past decade the present author has investigated extensively how the fundamental quantum-mechanical principles and properties affect the quantum computational performance of a quantum computer. Particular importantly, it is proven that there are the polynomial-time unstructured quantum search processes in quantum computation [56]. These works together form the research basis of this paper. This paper is mainly devoted to studying the mechanisms of quantum-computing speedup and especially exponential quantum-computing speedup of a quantum computational process (or a quantum algorithm). A quantum-computing speedup is determined only by the fundamental quantum-mechanical principles and properties of a quantum system. It is independent on purely mathematical logic principles of a problem to be solved on the quantum system. However, any one of these fundamental quantum-mechanical principles and properties that is responsible for speeding up a quantum computation has to obey these mathematical logic principles. This results in that the interaction is inevitable between the mathematical logic principles and the fundamental quantum-mechanical principles. Such an interaction could have an essential effect on the quantum computational process to solve the problem. These facts reflect importance of the interaction to affect quantum computational performance. At this point quantum computation is essentially different from classical computation.

One of the mostly important results in the paper is to show on the basis of the polynomial-time unstructured quantum search processes that the unitary quantum dynamics in quantum mechanics is the universal quantum driving force to speed up a quantum computation. A new quantum-computing speedup theory therefore is set up on the basis of the unitary quantum dynamics in a universal quantum computational model. It should be pointed out that both the reversible classical computational models and the quantum Turing machine models together are not sufficient to set up the theory. Both the unitary quantum dynamics and the symmetric structure and property of the Hilbert space of a quantum system are mainly responsible for an exponential quantum-computing speedup for an efficient quantum algorithm. The inherent importance for the unitary quantum dynamics to speed up a quantum computation lies in the unique ability of the unitary quantum dynamics to build the effective interaction between the symmetric structure and property of the Hilbert space of a quantum system (or more generally the fundamental quantum-mechanical principles) and the mathematical-logic principles including the mathematical symmetric structure of a problem to be solved on the quantum system. This unique ability is not owned by the reversible classical mechanics and the reversible equilibrium-state thermodynamics. It may generate an essential difference of computational power between quantum computation and classical computation with the aid of the symmetric structure and property of the Hilbert space. This effective interaction is of crucial importance for a general quantum algorithm to achieve an exponential speedup. In theory and experiment this effective interaction may be built up with the help of the unitary manipulation on the mathematical-logic functional operations of the problem.

It is well known that the classical computation has the three computing resources: (a) time or computational step number, (b) space or memory, and (c) energy or precision. Quantum computation also has these three computing resources. However, beside these three computing resources quantum computation also has the computing resource that is not owned by the classical computation. *This quantum-computing resource is the symmetric structure of Hilbert space of a composite quantum system.*

The old quantum-computing speedup theory is based on the quantum parallel principle, while the latter is based on the quantum superposition principle in quantum mechanics. Its underlying quantum-mechanical basis has been extensively considered as the quantum entanglement states which are closely related to the so-called quantum nonlocal effect. According to the new quantum-computing speedup theory the existing (efficient) quantum algorithms that are constructed in the frame of the old quantum-computing speedup theory are not full quantum. These quantum algorithms, which include the Abelian HSP quantum algorithms, non-Abelian HSP quantum algorithms, and conventional quantum search algorithms and so on, have the common character that the symmetric structure and property of the Hilbert space of the quantum system to perform these quantum algorithms does not have any effective effect on these quantum algorithms. This could be the main reason why these efficient quantum algorithms are quite special and considered to be semiclassical.

Since the early 2000s the unitary quantum dynamics has been considered as the fundamental and universal principle that is responsible for speeding up a quantum computational process in a physical system which may be a closed pure-state quantum system or a closed quantum ensemble. Now this hypothesis receives a strong support from the quantum-computing speedup theory. It is based on the assertion that only the unitary quantum dynamics in quantum mechanics is the universal quantum driving force to speed up a quantum computation. This assertion really supports strongly that the unitary quantum dynamics is the fundamental and universal principle not only in a closed pure-state quantum system but also in a closed quantum ensemble. On the other hand, it has been suspected extensively that the unitary quantum dynamics is a fundamental principle to describe a non-equilibrium physical process in a closed macroscopic physical system. Therefore, there is an apparent conflict between the unitary quantum dynamics and a non-equilibrium physical process in the physical system. How to resolve this apparent conflict is a great challenge in physics. In Ref. [37] the present author attempted for the first time to apply the quantum-computing speedup theory (or its related idea) to describing a non-equilibrium physical process of a closed physical system in the frame of the unitary quantum dynamics. This idea was suggested first in a quantum spin system [37] and then in an atomic physical system that has both the center-of-mass motion and internal motion [30, 41]. However, solving ultimately this great challenge problem has a long way to go.

The fundamental and universal principle that *both a closed quantum system and its quantum ensemble obey the same unitary quantum dynamics* could be very helpful for understanding essentially a large number of biophysical and biochemical processes in nature. It is well known that a protein folding process is an ultrafast physical process in nature. A classical search process generally can not match up the natural protein-folding process with such an ultra-high folding speed. In contrast, the polynomial-time quantum search process shows that such an ultrafast physical process is understandable according to the quantum-computing speedup theory. Moreover, the mechanism for exponential quantum-searching speedup predicts that *such an ultrafast natural process that does not obey the unitary quantum dynamics could be governed by the unitary quantum dynamics*.

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that a nuclear spin ensemble obeys the same unitary quantum dynamics as its pure-state counterpart does. It is well known that a nuclear spin ensemble with negligible spin relaxation effect obeys the unitary quantum dynamics (See, for example, the NMR book [46a]) and so does an electronic spin ensemble (See, for example, the ESR book [46b]).

34. From the point of view of the unitary manipulation a space-independent quantum system is usually a discrete quantum system that has only one manipulating freedom degree of the discrete internal motion of the quantum system. A spin system is a typical space-independent quantum system. On the other hand, a single atom motioning in time and space may have two independent manipulating freedom degrees of the discrete atomic internal motion and the continuous or discrete atomic center-of-mass motion. Therefore, a motional atom may not be a space-independent quantum system.

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36. X. Miao, *Universal construction of unitary transformation of quantum computation with one- and two-body interactions*, <http://arxiv.org/abs/quant-ph/0003068> (2000)

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38. N. J. Cerf, L. K. Grover, and C. P. Williams, *Nested quantum search and NP-complete problem*, Phys. Rev. A 61, 032303 (2000)

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41. X. Miao, *The STIRAP-based unitary decelerating and accelerating processes of a single free atom*, <http://arxiv.org/abs/quant-ph/0707.0063> (2007)

42. X. Miao, *Unitarily manipulating in time and space a Gaussian wave-packet motional state of a single atom in a quadratic potential field*, <http://arxiv.org/abs/quant-ph/0708.2129> (2007)

43. X. Miao, *Unitary manipulation of a single atom in time and space — Spatially-selective and internal-state-selective triggering pulses*, unpublished

44. (a) J. M. Myers, *Can a universal quantum computer be fully quantum?*, Phys. Rev. Lett. 78, 1823 (1997); (b) N. Linden and S. Popescu, *The halting problem for quantum computers*, <http://arxiv.org/abs/quant-ph/9806054> (1998); (c) M. Ozawa, *Quantum Turing machines: local transition, preparation, measurement, and halting*, <http://arxiv.org/abs/quant-ph/9809038> (1998); (d) Y. Shi, *Remarks on universal quantum computer*, <http://arxiv.org/abs/quant-ph/9908074> (1999); Phys. Lett. A 293, 277 (2002)

45. These conflicts do not exist in the universal quantum circuit model [5, 44]. Strictly speaking, they may be considered to be outside the universal quantum computational models under study at present in the sense that they really

involve in the quantum measurement. In quantum measurement a quantum system should be considered as an open system and hence its time evolution process generally does not obey the unitary quantum dynamics. This is not only because the quantum measurement results in collapse of the wave function of the quantum system but also because there is a loss of information of the quantum system to outside the system in the quantum measurement.

46. (a) R. R. Ernst, G. Bodenhausen, and A. Wokaun, *Principles of Nuclear Magnetic Resonance in One and Two Dimensions*, Oxford University Press, Oxford, 1987; (b) A. Schweiger and G. Jeschke, *Principles of Pulse Electron Paramagnetic Resonance*, Oxford University Press, 2001

47. L. D. Landau, E. M. Lifshitz, and L. P. Pitaevskii, *Statistical Physics (Part 1)*, 3rd edn, translated by J. B. Sykes and M. J. Kearsley, Pergamon Press, New York, 1980

48. That the mechanism for exponential quantum-searching speedup is generally available does not mean that an exponential speedup always can be achieved in a physical or computational process. However, if a physical or computational process that is as hard as an NP-complete problem takes place in polynomial time, then it is certain that the mechanism exists in the process.

49. (a) A. Ekert and R. Jozsa, *Quantum algorithms: entanglement-enhanced information processing*, Phil. Trans. Roy. Soc. Lond. A 356, 1769 (1998); (b) R. Jozsa and N. Linden, *On the role of entanglement in quantum computational speed-up*, <http://arxiv.org/abs/quant-ph/0201143> (2002); (c) E. Knill and R. Laflamme, *On the power of one bit of quantum information*, Phys. Rev. Lett. 81, 5672 (1998); (d) S. Parker and M. B. Plenio, *Efficient factorization with a single pure qubit and  $\log_2 N$  mixed qubits*, Phys. Rev. Lett. 85, 3049 (2000); Also see: <http://arxiv.org/abs/quant-ph/0102136> (2001)

50. (a) S. L. Braunstein, C. M. Caves, R. Jozsa, N. Linden, S. Popescu, and R. Schack, *Separability of very noisy mixed states and implications for NMR quantum computation*, Phys. Rev. Lett. 83, 1054 (1999); (b) T. M. Yu, K. R. Brown, and I. L. Chuang, *Bounds on the entanglability of thermal states in liquid-state nuclear magnetic resonance*, Phys. Rev. A 71, 032341 (2005)

51. (a) M. J. Bremner, C. Mora, and A. Winter, *Are random pure states useful for quantum computation?*, <http://arxiv.org/abs/quant-ph/0812.3001> (2008); (b) D. Gross, S. T. Flammia, and J. Eisert, *Most quantum states are too entangled to be useful as computational resources*, <http://arxiv.org/abs/quant-ph/0810.4331> (2008); (c) M. Van den Nest, *Classical simulation of quantum computation, the Gottesman-knill theorem, and slightly beyond*, <http://arxiv.org/abs/quant-ph/0811.0898> (2008)

52. Here gives a testing program for the quantum-computing speedup of a quantum algorithm. For a given efficient quantum algorithm according to the qubit number, initial state, output state, and running time one may construct a purely quantum-mechanical (QM) unitary sequence which is equivalent to the quantum algorithm. Different qubit number may lead to different initial state, output state, running time, and QM unitary sequence. The testing program is carried out by two experimenters, in which the quantum algorithm is executed in a black box. Experimenter One is responsible for preparing the initial state and

measuring the output state, while Experimenter Two is responsible for putting the quantum algorithm or its equivalent QM unitary sequence into the black box and setting the running time. Experimenter Two need not tell Experimenter One which one of the quantum algorithm and its equivalent QM unitary sequence is put into the black box. For a given qubit number Experimenter One prepares the initial state and then inputs it into the black box. Experimenter Two then may put the quantum algorithm or its equivalent QM unitary sequence into the black box. After the setting running time Experimenter One measures the output state. If she or he finds that the output state is the desired state, then she or he believes that the quantum algorithm indeed achieves the desired quantum-computing speedup. This process may be repeated many times with different qubit numbers, initial states, and running times till the final result is obtained. Since sometimes Experimenter Two may put a purely QM unitary sequence into the black box, this leads to that a wrong conclusion that the purely QM unitary sequence also has an exponential speedup to solve the same problem is obtained by Experimenter One from the measured result. This is no sense! What is the reason? Because Experimenter One believes that only the quantum states themselves are responsible for a quantum-computing speedup, she or he need not care about which one of the quantum algorithm and its equivalent QM unitary sequence is put into the black box, and what she or he needs to care about is the input states and output states. This special testing program above may be extended to a general case. As long as Experimenter One does not consider explicitly every mathematical logic operation of the efficient quantum algorithm, she or he could possibly obtain a similar wrong conclusion.

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54. An exponentially large space means that dimensional size of the space increases exponentially with the qubit number of quantum system (or problem size). A polynomially small space means that dimensional size of the space increases polynomially with the qubit number.

55. E. Farhi and S. Gutmann, *An analog analogue of a digital quantum computation*, Phys. Rev. A 57, 2403 (1998) or <http://arxiv.org/abs/quant-ph/9612026> (1996)

56. X. Miao, *a polynomial-time unstructured quantum search process based on the unitary quantum dynamics and Hilbert-space symmetric structure*, unpublished